ADVANCES IN THE FOUNDATIONS

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1. Introduction

This catchy (and ambiguous) title occurred to me when I realized (1) I would not be able to attend the ANPA 11, (2) I did not know how much time I could devote to writing a paper for presentation in my absence, and (3) I could not be certain of which advances could be discussed nor how mature they might be. Having thus set your expectations I proceed.

It is my desire to convey the directions which my research into the foundations of discrete physics has taken since ANPA 10. Under the circumstance of little time, these are largely conceptual directions although some results are so obvious to me that they may be convincing to others as well. The topics which I wish to outline are three:

1. A new presentation of the "adding in quadrature" formula (which appears in my paper "The Fine Structure of Hydrogen").
2. An explanation of how to apply the same second order correction methods for other computations, especially the mass of the pion and the value of the weak angle.
3. A new way of making contact with QED via Feynman's Path Integral formulation.

2. Adding In Quadrature: Why?

Over the last year, the one aspect of my computation of the second order correction to the fine structure constant which was most difficult to explain was what Noyes has referred to as adding in quadrature. As I have pointed out, adding in quadrature is actually a consequence of having what I think of as simultaneous and independent "paths" in the system. In Feynman's terminology these are alternative "paths". My own, largely non-verbal, conception of the systems involved has finally found expression in terms of a certain complex system and the frequency with which certain alternatives will be manifested.

The terminology I shall use is that of mathematical probability (see Uspensky). This is important to remember since the meanings of the terms in the physical sciences can be quite different. So as not to burden the reader with having to look up the definitions of terms in the references, I will define a few key ones here.

By an *event* I mean a well–defined, abstract arrangement or class of arrangements. The event (arrangement) may manifest in either space or time or both. An arrangement is identified by its properties or an equivalence class of properties. In fact, a manifestation of an event may be purely abstract. It may be defined only in some mathematical context and so be not physical at all. I will use the term *statistical event* when I mean this kind of event, since this differs from physical events.

By an *occurrence* of a statistical event I mean a particular manifestation of that statistical event. Each particular way (one detailed arrangement among the arrangements of an equivalence class of arrangements) in which the statistical event can be manifested is called a *case*. For example, the results of throwing a die is an occurrence and each of the possible results (a face with 1, 2, 3, 4, 5, or 6 points) is a case. The number of ways in which a case can occur is called a *case count*—these are the number of favorable cases for a particular outcome.

The confluence of one or more statistical events will be said to form a *system*. Exactly how these multiple statistical events are arranged in the system also defines

a statistical event, albeit at a higher level of complexity. I will refer to the partitions of the system, each consisting of a component statistical event as the *sub–systems.*

The system under investigation consists of two or more statistical events. For convenience, I will speak of only two. These two events can occur in various ways, called the cases for each event. A particular set of possible cases provides a *representation* of the sub–systems; any given representation may or may not be orthogonal and complete in the sense that it allows us to distinguish (a) each of the cases and (b) the sub–systems.

We can think then of a particular manifestation of a statistical event as sampling from a population consisting of p copies of all the possible cases. The populations (in particular, the individual cases) for two or more statistical events are not necessarily distinct. The information available about the system is (1) the number of (not necessarily distinct) members of each population, (2) the relative frequency with which each case occurs, and (3) the representation system.

For example, we might know that there are 2^q cases and q^2 possible labels to represent those cases for each sub–system. The problem is then as follows: Is there a way of representing the system in terms of the 2^q cases with the q^2 labels (*i.e.* a map) of each sub–system such that (1) the maximum amount of information about an event in the total system is obtained and (2) any known constraints regarding relative frequencies, cases, or populations are respected?

The answer is sometimes yes, given information about the structure of the system. Following a constructive point–of–view, we are required to construct the statistics about the system from the cases for each of the statistical events.

Consider two statistical events A and B. Let these two statistical events jointly have a total of m cases (which we write as $(a)+(b)$) subject to two conditions: (1) The statistical events A and B are *statistically equivalent* in the sense that there is a mapping for which pairwise mappings of cases have the same case count. (2) They are also *independent*, however, in the sense that the (a) and (b) cases are distinguishable, *i.e.* assignable to A and B respectively, except for n cases. These

n cases must be assigned to either A or B arbitrarily.* For example if $n = 1$, we do not know whether A consists of either $(\frac{m}{2} - 1)$ or $(\frac{m}{2} + 1)$ distinguishable cases. Thus, whenever one of the n indistinguishable cases occur, we must recognize that a statistical event of both types (*i.e.* A and B) has manifested. So in this sense, n is a measure of the degree to which A and B are not independent. Note that the number of cases *must* be non-integral for odd numbers of cases. However, this is not a problem if, as here, we have assumed that A and B are statistically equivalent in terms of the number of case counts (condition (1) in the preceeding paragraph). If A and B are not statistically equivalent, then we do not simply divide m by 2 in the formula.

The necessity of assigning the n cases to A or B is a problem if we assume the independence of A and B. When we count cases, there is an implied assumption that we know what we are counting cases of ... that the cases are classifiable into those which should be counted for A and those which should be counted for B. We have no place for indistinguishable cases, and so must find a way to assign the n cases to either A or B. Since we know m and we know that A and B are identical except for the n cases, we know that if we assign n to either A or B, then $(\frac{m}{2} - n) + (\frac{m}{2} + n) = m$, and then the value of (a) is $(\frac{m}{2} - n)$ and that of (b) is $(\frac{m}{2} + n)$ or vice–versa.

As an example, consider a special die. The faces on this die are either green or blue with the exception of n faces. These n faces are turquoise. An omniscient observer can distinguish green and blue from turquoise. Our real observer is not even aware that the color turquoise exists and so always sees either green or blue, even when the face is turquoise. In this example, I assume that there is no particular bias toward either green or blue and so the real observer says a turquoise face is either green or blue with equal probability. This assumption of equal a

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$\star$ Note that n is integer unless A and B are periodically repeating statistical events. Then it is possible that x cases out of every y repetitions of A and/or B are indistinguishable. Then we may take $\frac{x}{y}$ as the expectation value of n. If A and B are not strictly repeating, the relationship must be analyzed more carefully and the expectation value of n must be computed accordingly.

priori probabilities is the only reasonable one when there is no information to the contrary.

Suppose we have one such die so that there are 6 faces total with one turquoise face on the die, three of green (or blue) and two blue (or green). A green face is a favorable case for the statistical event A and a blue face is a favorable case for the statistical event B. Our real observer assumes that the die contain no turquoise faces and in fact that green and blue are equally distributed. Whenever our real observer throws a die, if the turquoise face comes up, it can be identified as either green or blue. If it is identified as blue, this simply re–establishes the statistical equivalence of A and B with $(a) = (b) = \frac{m}{2}$. A and B are treated then as completely independent. Note that this is equivalent to there being no turquoise faces at all. Otherwise one obtains $(\frac{m}{2} + 1)$ for (a) and $(\frac{m}{2} - 1)$ for (b).

We can made the situation more interesting, if less physical, by using two dice, one of which has a turquoise face and both of which are otherwise either totally green or totally blue. In this system, the turquoise face couples A and B. This couples the two dice when the judgment call is "bad"—*e.g.* identifying as green a turquoise face on an otherwise blue face die. When the judgment call is "good", each of the two die always contributes to either statistical event A or statistical event B (but never to both) and so they are independent in the context of A and B.

If there is no way of telling whether or not the turquoise face is on the "green die" or the "blue die", this situation corresponds to the kind of system described in the computation of the fine structure constant.[⋆] In the physical experiment, we are always looking for data regarding a coupled system (*i.e.* an "orbit" composed of two oscillations, one corresponding to the major axis and the other to the minor axis of an ellipse. By definition, the judgment call is always "bad". When the call is "good", we obtain a system that is partitionable into two statistically equivalent

⋆ In the ordering operator calculus, the attribute distance between the two turquoise faces can be zero so that they are truly indistinguishable. This kind of coupling is pre–supposed in the analysis of the fine structure constant.

and completely separable sub–systems. We would call the results *noise* in the experimental system and throw out the data. Such data comes from the case where the "orbit" is circular.

To continue, for event A, the probability of one of the (a) cases manifesting is then $\frac{1}{(m/2)} * (\frac{m}{2} - n)$ and similarly for event B. We write these as (A') and (B'), where (A) would be the true probability of an A case were we able to deal with the partial independence of A and B. Under the assignment of n to either A or B, the events are once again independent. The compound probability (AB) of simultaneous occurrence of cases belonging to A and B is then

$$(A')(B') = \left[\frac{1}{(m/2)}\right]^2 \left(\frac{m}{2} - n\right) \left(\frac{m}{2} + n\right)$$

Given such a compound probability, under the (real observer's) assumption of complete identity and independence of A and B, it is natural to then compute the $(A) = (B)$ as

$$(A)^2 = [(A') * (B')]$$

This, on expansion and rearrangement give the "adding in quadrature" formula"

$$\left(\frac{m}{2}\right)^2 (A)^2 + (n)^2 = \left(\frac{m}{2}\right)^2 .$$

However, the physical way in which (A') and (B') are measured in a physical system may determine whether or not both (A') and (B') contribute. For the fine structure constant measurement, it is only (A'), corresponding to $(\frac{m}{2} - n)$ that contributes to the measured compound probability:

$$(A) = [(A') * (A')]^{1/2}$$

In the case of the computing the fine structure constant, we have $m = 4*127*15$ and $n = 1$. The factor of four comes from two independent events having $127 * 15$

cases each, any pair of which may be ordered as A then B or B then A, *i.e.* in two more ways.† Each repeated occurrence of the compound event A and B is a mutually exclusive case and it takes 137 such cases to have a Coulomb event. The total probability for a set of mutually exclusive and independent cases is then just

$$137 * \alpha = \left[\frac{1}{(m/2)}\right] (A')$$

But (A') is just $(\frac{m}{2} - 1) = (2 * 15 * 127 - 1)$, so that

$$\alpha = \left(\frac{1}{137}\right)\left[1 - \left(\frac{1}{2 * 15 * 127}\right)\right]$$

The two statistical factors contribute to the combined event A AND B. By multiplying the (almost) independent probabilities, one obtains a formula for "adding in quadrature" on rearrangement of the terms. The formula is, of course, different if there are more than two almost independent events.

To obtain the general case, one must consider more than two statistical events (of number k) and all the various possibilities for judging the indistinguishable cases as being favorable to one or more of these statistical events. All possible assignments of the n indistinguishables must be taken into account. Whenever n is a multiple of k, there is the possibility of an equi-probable assignment to all k statistical events. Otherwise one must obtain terms in the relative frequencies which look like binomial coefficients. I hope to have an opportunity to spell out the general case in the near future.

† Note that in the general case of p multiple events the factor of two is replaced by the number of permutations of p events.

3. Other Second Order Corrections

The currently accepted empirical value of the fine structure constant is 1/(137.035963(15)). According to the combinatorial hierarchy and Program Universe, the calculated value to first order would be 1/137. Note that this is the value obtained if (A') is $\frac{m}{2}$—*i.e.* if $n = 0$ and events A and B are in fact identical. When the structure of A and B, and the possibility that they are not independent $(n = 1)$, are taken into account, this yields a second order correction. The value obtained is given by

$$\left(\frac{1}{137}\right)\left(1 - \frac{1}{2 * 15 * 127}\right) = \frac{1}{(137.0359674)}$$

in close agreement with the empirical value.

By following an argument directly analogous to that presented in computing the fine structure constant, one can obtain corrections to the weak or Weinberg angle and to the Fermi coupling. This should not be surprising since there is a relationship between the "weak" structure constant and the fine structure, as introduced by Glashow:

$$g_W = \frac{e}{\sin\theta_W}$$

Thus, given that the definition of the fine structure constant is $e^2/\hbar c$ and weak structure constant analogously by $g_W^2/\hbar c$, we have the ratio of the fine structure constant to the weak structure constant:

$$\frac{a}{a_W} = \sin^2\theta_W$$

The weak coupling g_W and the Fermi coupling G_F are related by

$$G_F = 2^{1/2} g_W^2 r_W^2$$

where r_W is the range of the weak force.

Just as the Coulomb event depends on 127 cases of level 3 of the combinatorial hierarchy and the 16 - 1 possible labels to represent them, the weak event depends on the 7 cases of level 2 and the 4 - 1 possible labels to represent them. This gives the number of cases as $3 * 7$ analogous to $15 * 127$. However, the measurement of the weak event is not in the context of a bound system like that of the hydrogen atom, but rather is understood in the context of a decay process. There is no reference frame in which to distinguish event A from event B. As a result, the cases for each of two events are not orderable: the factor of two that occurs in $\frac{m}{2}$ for the Coulomb event does not occur for the weak event. Thus $\frac{m}{2}$ is $3 * 7$ and the correction term (A') is $(1 - \frac{1}{3*7})$.

Fermi Coupling Constant

The empirical value of the Fermi coupling (as given in terms of the proton mass and factoring out the square root of two and the proton mass squared that would otherwise appear) appears as

$$G_F * (2)^{1/2} = 1.02684(2) * 10^{-5} .$$

To first order, this value is calculated from the combinatorics of Program Universe as

$$\frac{1}{(256 * 256)} = 1.07896 * 10^{-5}$$

The second order correction gives

$$(1.07896 * 10^{-5}) \left(1 - \frac{1}{3 * 7}\right) = 1.0275808 * 10^{-5}$$

again in close agreement with the empirical value.

The Weak Angle

The currently accepted weak or Weinberg angle squared empirical value is

$$\sin^2\theta = 0.229(4)$$

Again, according to Program Universe the ratio of weak to Coulomb events is 2:1 so that the calculated first order value of the weak angle is 1/2 and the square of the weak angle is then

0.25.

The second order correction applied to the weak angle (not to the square) and then squaring gives

$$\left(0.5 * \left[1 - \frac{1}{3 * 7}\right]\right)^2 = 0.2267573$$

again in good agreement with the empirical value.

Note that we have no "running constants" in our theory nor do we have perturbative approximations. Ours are true corrections due to well–defined, finite system effects. We do anticipate a proper correction factor correlated with the energy of the system.

4. Some Further Speculations

Having had reasonably good success with computing these physical values, I am inclined to make a few conjectures under the assumption that similar corrections would work for other physical values. Three have been presented by Program Universe to–date: the charged pion/electron mass ratio, the neutral pion/electron mass ratio, and the gravitational structure constant.

Charged Pion/Electron Mass Ratio

The empirical value of the charged pion/electron mass ratio is

273.13

The pion is represented in Program Universe as 137 electron positron pairs plus either an electron—antineutrino or a positron–neutrino pair, suggesting a mass ratio of

275.

First note that this is a bound system of two sets of 137, *i.e.* electron type events and positron type events. Thus ordering is important and the factor of 2 discounted in the correction of the weak structure can not be discounted here. Suppose that the charged pion electron/positron pairs are weakly coupled ($3 * 7$ cases) via the 7 labels required to represent level 2. Furthermore, suppose that there are two cases (via exchange of weak labels) which can not be distinguished as belonged to either the electron or the positron set (*i.e.* $n = 2$) for each pair.

The second order correction is then

$$275 * \left[1 - \frac{2}{2 * 3 * 7 * 7}\right] = 273.12925$$

in excellent agreement with the empirical value.

Neutral Pion/Electron Mass Ratio

The empirical value of the neutral pion/electron mass ratio is

264.10.

Program Universe suggests that the pion consists of 137 electron/positron pairs, so that the first order computed value is

274.

Suppose the neutral pion is a more complicated system. If there are three indistinguishable cases instead of two, and only manifesting half the time, and

that it does not couple back to the 7 labels in level 2 as the charged pion does, the second order correction is*

$$274 * \left[1 - \frac{3}{2 * 2 * 3 * 7}\right] = 264.21428$$

Gravitational Structure Constant

The empirical value of the gravitational analogue to the fine structure constant is

$$\hbar c/Gm^2 = 1.6937(10) * 10^{38}$$

where m is the proton mass.

The first order value computed by Program Universe is

$$(2^{127} - 1 + 137) = 1.70147 * 10^{38} .$$

Suppose that there is a coupling between a combinatorial hierarchy level 2 weak event with $3 * 7$ cases and a compound level 1 - level 2 event with $3 + 7 = 10$ cases. These then couple to give $3 * 7 * 10$ possible cases. If one of these cases is indistinguishable ($n = 1$) and order is unimportant, one obtains for a second order correction

$$(1.70147 * 10^{38}) \left[1 - \frac{1}{3 * 7 * 10}\right] = 1.6933675 * 10^{38}$$

again in good agreement with the empirical value.

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* Compare this with Noyes recent argument in "Bit String Scattering Theory" – SLAC PUB 5085, January 16, 1990 – from estimation of the pion-nucleon coupling constant.

5. Path Integrals and Event Networks

A short while ago Karmanov suggested (correspondence to Noyes) that there was a close correspondence between the random walk approach to quantum mechanics of Stein and the path integral approach of Feynman. He pointed to Problem 2-6 worked out by Feynman and Hibbs which is a discrete one-dimensional path integral yielding the appropriate amplitude and kernel for a relativistic free particle moving in one dimension, equivalent to the Dirac equation. He later referenced Jacobson and Schulman.

At about the same time, I was working on the problem of interconnecting Noyes bit string events to provide a global causal structure. It has long been my belief (coming from a general relativistic or geometrodynamic point-of-view) that 4-space events are the observables which should define the space-time causal structure, rather than assuming a space-time causal structure in which observable events occur. Since Program Universe provides a mechanism for generating "events", only some of which have physical significance, we can generate a collection of such physically interpretable events. The question then arises "how shall these events be seen as interconnected?".

A clue to this is provided in FDP in the discussion on persistent objects. The idea is that a physical system is identifiable over its evolution if its properties are conserved. To cast this another way, we say that a set of events describes a single system or object if all the events in the collection have the same set of properties. At the quantum level, these properties are specified by the conservation laws and the quantum numbers. If the system is understood to be evolving in time, then there must be an ordering to the events—we must be able to state a criterion by which we say that one event is later than another. These ideas are explored a bit further in "The Fine Structure of Hydrogen".

This suggests an event network, defined by the combinatorics of the bit-string events themselves. Nodes in the network denote events. Arcs denote connectivity between events. The arcs are directed. The events which can be interconnected

are constrained by the conservation laws—not because of any intrinsic factor—but because without them no property persists and events can not be ordered into a causal structure.

The event network is characterized by ordering in terms of the generating operator (call this Tick) and by units of "action". The first is NOT physical time, but is probably related to it. In particular, the complexity of the network increases with Tick. Given the event network, a key problem is how to translate from this "Tick–action" space into a space–time causal representation.

This problem has been only partially solved to date*. In the event network, two connected events may be Tick ordered. However, an increment in the action can not occur without a Tick. On the other hand a Tick can occur without an increment in the action. Events which are connected in this way are indistinguishable in the event network except by their connectivity.

In order to research Karmanov's comments, I read Feynman and Hibbs "Quantum Mechanics and Path Integrals". It quickly became clear that path integrals are formed over event networks in a continuum space–time representation. Here the difficulty is the opposite of mine; "how does one constrain the numbers of possible events so that the expression for the probability amplitude does not diverge?" This appears as an infinity of possible paths, related to the others by a phase factor. In the convergent cases, the phase serves to diminish the significance of the contribution of most of the paths. It does this in two ways: (1) by there being a nearby path that cancels out the contribution of the path under consideration, (2) by reducing the contribution to the amplitude directly so that for paths at infinity the contribution is zero.

Consider a discrete version of the Feynman path integral. Each point along the paths has a particular action as is clear from the Lagrangian form of the action integral which Feynman uses to define the amplitude. Clearly it is possible to

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⋆ September, 1989

translate a Feynman path integral formulation into an event network in Tick–action space. Of course, this is a backwards approach from my perspective, but it does show that the event network in Tick–action space can be used to solve problems that are solvable by Feynman path integrals.

The classical version of the event network for a free particle leads immediately to the correct expression for the dynamics. Furthermore, the classical event network inherently obeys the principle of least action.

This deserves some explanation. A particle or object is said to follow a *continuum classical path or trajectory* if, given two space–time points on the path, there is always a third point "between" them such that the particle will be found at the third point. In other words, any other path would not preserve the unique identity of the "particle". There always exists a path through the network which consists of the least action and this path also has the fewest indistinguishables. Of course, our network is discrete. The definition is suitably modified so that the "classical path" is not infinitely divisible.

A similar definition is important in understanding what is meant by *interacting particles* in an event network. Suppose that two particles are identifiable at entry to the network and also at the end (exit) of the network. If it is possible to define a classical path for each of the particles on the event network, then they are *non-interacting*. Otherwise, they are interacting. Note that for the portion of the network where two particles interact, they lose their identity—they are not separable in terms of the space–time causal structure. This definition is consistent with our definition of particle (or object) as a conceptual carrier of properties between events.

This definition is also consistent with Feynman's definition of interacting particles for the path integral formulation of quantum mechanics and QED. However, there is an important difference between the classical network and the (relativistic) quantum network. In particular, the allowed paths which preserve the properties of the particle may go backwards in time. In addition, indistinguishable events

contribute to the amplitude, whereas in the classical network they do not. The reason for these differences is straightforward: there need not be a unique path in the event network which serves to propagate all the properties of the particle—it may be propagated by multiple paths. This does not imply that multiple particles will always interact in the quantum network.

The last conceptual point to be made is that the event network is easily modified to take "potentials" into account. This modification effectively results in changing the density in space–time of the observable events. This is affected in the translation to a space–time causal representation of the network or a path on the network.

In solving any particular problem, the key is to develop the appropriate event network probability amplitude. First the network is constrained by the number of nodes that are possible. This is done combinatorially from the bit string representation of the properties involved in the system. The network is to be treated as a complex 4–event or 3–event. What takes place within the event is unknown from the outset, but can be solved according to the conditions on the input and output. These limit the possible kinds of nodes that can be generated from Program Universe. In particular, due to the conservation laws, directly observable new properties can not arise within the event complex.

At this point I am working to define how one computes the number of paths within the network if the number of nodes that can be generated under the constraints is n. The following is known straight–away:

1. The number of possible arcs in graphs consisting of n–nodes is $\frac{n^2}{2}$.
2. The number of directed graphs is twice that number.

We must then determine:

3. The number of directed graphs which are connected—*i.e.* there are no isolated subgraphs and every node either initiates or terminates a directed arc?
4. Of these directed and connected graphs, how many have at least one directed

arc initiated at each of the "entry nodes" and at least one directed arc terminating at each of the "exit nodes"? Each of these graphs represents a possible configuration for the interior of the event complex.

5. For each of these graphs, how many ways are there to walk the graph such that the walk begins on the "entry nodes" for the network and terminates on the "exit nodes"? Each such walk corresponds to another way of generating an n-node graph representing the interior of the event complex, *i.e.* an ordering operator.

6. How many of these walks are indistinguishable from another such walk under some permutation of the node labels? This is essential to computing the amplitude: we need to know both how many unique walks there are and how these are weighted by the permutation of node labels.

I have not had the time to pursue these questions. However, I am fairly certain they are solved problems in graph theory. I am also fairly certain the answers can be cast in terms of combinations and permutations. Once they are solved and cast in this form, it is a simple matter to express the formula in terms of the discrete transport operator as defined in FDP and to draw the correspondence to the Feynman amplitude (which is expressed as a phase "transport" *).

I am confident that the event network formulation in "Tick-action" space is richer than the path integral formulation and that it provides a linkage between the bit string events of Program Universe and an accepted formulation of QM and QED. It comes to us fully relativistic and finite—there are no divergences nor need for renormalization.

$\star$ See FDP for a derivation of the transport operator.

6. Conclusion

In my absence from these pursuits, I invite others to contribute to these computational endeavors in order to speed us on.

7. Postscript

Since this paper was written, these ideas have been applied to the finite and discrete construction of the $1+1$ Dirac equation in such a way that the finite step length is preserved. The method appears to apply to the $2+1$ and $3+1$ problem as well.

In a letter dated November 1, 1989, Karmanov has written that the finite step length does not survive when computing the $1+1$ Dirac (or $2+1$ or $3+1$) from a Stein random walk model (along the lines of Feynman and also Jacobsen and Schulman). However, it turns out that the derivation of the amplitude assumes that the number of trajectories is given by the binomial coefficient and this was then approximated as (number of right [or left] turns) to the k power, divided by k factorial for large N. This approximation is wrong if the step length is fixed. I have pointed out (November 28, 1989) that the number of trajectories should be given by my expression for the "total attribute distance" (derived in FDP) since in our model all "turns" are indistinguishable, there being no *a priori* coordinate system to distinguish them. This situation is similar to the more familiar case of "spin flips". The total attribute distance is just the needed formula arrived at by Karmanov earlier by an approximation in the large N limit, but is now precise and independent of N. The Dirac equation follows and the finite step length survives.

8. References

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4. McGoveran, D., The Fine Structure of Hydrogen, version dated March 17, 1988, Proceedings of ANPA 10, pp.28-52.

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